Sem:- IV

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Unit-2

MJC PHY 05

Linear Differential Equation

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A linear differential eqⁿ is defined as a differential eqⁿ of the type

$$(D^n + P_1 D^{n-1} + P_2 D^{n-2} + \cdots\cdots + P_n)y = Q$$

where $D = \frac{d}{dy}$, $D^2 = \frac{d^2}{dy^2}$, ...etc and $P_1, P_2 \ldots, P_n, Q$ are functions only of x.

If atleast one of $P_1, P_2 \ldots P_n$ or Q involves y, it is no longer linear.

The most general form of the linear differential eqⁿ. ~~The most~~ of first order is

$$\frac{dy}{dx} + Py = Q$$

where P & Q are functions of x.

Method of Solution of Equation

$$\frac{dy}{dx} + Py = Q$$

Multiply both sides by $e^{\int P dx}$, we get

$$e^{\int P dx}\frac{dy}{dx} + P e^{\int P dx} y = Q e^{\int P dx}$$

or $$\left(P + \frac{d}{dx}\right)\left(y e^{\int P dx}\right) = Q e^{\int P dx}$$

Integrating with respect to x,

$$y e^{\int P dx} = \int Q e^{\int P dx} dx + c$$

where c is an arbitrary constant.

* The factor $e^{\int P dx}$ is called integrating factor.

Equations Reducible to Linear Form

$$\frac{dy}{dx} + Py = Qy^n$$ is not linear.

After dividing both sides by y^n.

$$y^{-n}\frac{dy}{dx}\ y^{1-n} P = Q$$

Let $z = y^{1-n}$

$$\Rightarrow \frac{dz}{dx} = (1-n) y^{-n} \frac{dy}{dx}$$

The differential eqn becomes

$$\frac{1}{(1-n)}\frac{dz}{dx} + Pz = (1-n)Q$$

which is clearly a linear eqn and can be solved by using the integrationg factor $e^{\int (1-n) P dx}$.

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